TECHNICAL NOTES

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No. 306

CURVES SHOWING COLUMN STRENGTH OF STEEL

AND DURALUMIN TUBING

By Orrin E. Ross

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The following set of column strength curves are intended to simplify the method of determining the size of struts in an airplane structure when the load in the member is known, and to simplify the checking of the strength of a strut, knowing the size and length.

In the past it has been customary to compute the size of a strut by trial and error, with the use of the basic formulas, or by the use of nomographic charts, which were conducive of error because of the necessity for referring to tables to be sure that the strut came within Johnson's or Euler's range.

With the following curves no computations are necessary, as in the case of the old-fashioned method of strut design; no straitedge is needed to connect points, as in the case of the nomographic charts; no reference need be made to tables to ascertain the limiting length of a strut, as the curve for each size strut is complete in itself through the range of long and short columns. The process is so simple that draftsmen or others who are not entirely familiar with mechanics can check the strength of a strut without much danger of error. If it is desirable to use the lightest tube available for the same strength

it can be seen at a glance which are the lighter, because the tubes ascend the ordinate of zero length with increasing cross-sectional area and hence greater weight. Therefore, if two or more tubes give the same strength for a given length, the lightest can be determined because its curve will intersect the ordinate of zero length in the lowest position. If a member (as in the case of some fuselage members) is designed by tension, the size can be determined from these charts if the yield point is used as the allowable tensile strength. The tensile strength is to be found on the ordinate of zero length.

The idea of plotting column strength curves in this fashion is not new, but it was not until recent years that standard sizes of tubing were decided upon, and to compile a family of curves for every known size would be a tremendous task.

Figure 1 is a set of basic column curves for steel. The curves are plotted to Euler's long strut formula

$$\frac{P}{A} = \frac{C\,\pi^2\,E}{(L/R)^2}$$

and Johnson's short strut formula,

$$\frac{P}{A} = f_c - \frac{f_c^{\,2}}{AC\,\pi^2\,E}\left(\frac{L}{R}\right)^2$$

where (in both formulas)

P = load in pounds,

A = cross-sectional area of strut,

C = fixity coefficient,

E = modulus of elasticity,

L = length of strut,

R = radius of gyration of strut cross section,

f_c = yield point of the material in compression in pounds per square inch.

Johnson's formula is for a parabolic curve and was devised by Mr. J. B. Johnson as a formula for short columns since short struts do not follow Euler's law in actual test. The point on Euler's curve where the columns begin to depart from it, is the point of tangency of the two curves and is on the ordinate where

$$\frac{L}{R} = \sqrt{\frac{2C\ \pi^2\ E}{f_c}}$$

These curves are used in plotting the curves of Figures 3 to 11, inclusive. They can be used for determining the strength of other steel columns with the same physical characteristics, but of various cross-sectional properties.

The fixity coefficient C as used in all the column strength curves herein is a constant which depends on the degree of restraint of the ends of the strut, that is, for a pin-ended strut $C = 1$ and for an absolutely fixed-ended strut $C = 4$. A theoretical derivation of this can be found in such standard textbooks as "Applied Mechanics," by Fuller and Johnson, and "Strength of Materials," by Morley. In airplane structures the fixity coefficient rarely exceeds two (2) and never reaches four (4). This has been determined by taking the results of a

large number of static tests and computing backwards to solve for the value of C. Therefore, in a welded structure such as a fuselage or a trussed wing beam it is not safe to use a value of C larger than 2.

Figure 3 is a family of column strength curves for mild carbom streamline tubes. These show the axial compression load (in pounds) which may be allowed on the various sections shown according to the length of the strut. The tubes are not standard throughout the United States, but are used extensively by at least one company. They are plotted only for a fixity coefficient of 1 because streamline tubes are usually pin-ended in practice.

Figures 4 and 5 are columm strength curves for mild carbon round steel tubes with fixity coefficients of 1 and 2, respectively. All tubes on these charts are taken from the standard list adopted by the airplane manufacturers and government departments of the United States. Table A gives the list of standard round steel tubes.

Figures 6 to 11 are column strength curves for high strength round steel tubes (either chrome-molybdenum or nickel steel). Note that Figures 10 and 11 are for steel tubes heat-treated to a yield point of 105,000 lb./sq.in. These tubes are only advantageous to use in short lengths because for tubes in Euler's range the strength is the same as for tubes not heat-treated. The curves for these tubes are plotted only for a fixity of 1,

since any welding or operation which tended to fix the ends would destroy the heat treatment.

In order that the method of plotting these strength curves may be better understood, Table B contains the computations for three struts on Figure 6 and is typical for all the curves in this report. Should it be desired to plot the curve for any other round or streamlined tube which is not on the standard list or for any other section such as a square, angle, channel or tee section, it is necessary to compute the cross-sectional area and the radius of gyration which is equal to the square root of the moment of inertia divided by the area or

$$R = \sqrt{\frac{I}{A}}.$$

Solving for L/R is a simple slide rule operation. When L/R is found for each increment of length desired, the corresponding value of P/A can be taken from Figures 1 or 2 (depending on the material used) and the value of P can be found by another slide rule operation by multiplying the value of P/A by the value of A. Caution must be exercised to use the proper curve on Figures 1 and 2 as regards the fixity of the strut to be computed.

Figure 2 is the basic column curves for duralumin. The curves are plotted to Euler's formula for long columns and the straight line formula for short columns

$$\frac{P}{A} = 48{,}000 - 400\ \frac{L}{R} \text{ for a fixity of } C = 1.$$

$$\frac{P}{A} = 48{,}000 - 280\ \frac{L}{R} \text{ for a fixity of } C = 2.$$

The symbols are the same as those for Euler's and Johnson's formulas. The straight-line formulas are purely empirical and are based on tests conducted by the Materiel Division of the Army Air Corps. The point of tangency of the straight line and Euler's formula is approximately on the ordinate where

$$\frac{L}{R} = 66.6 \text{ for } C = 1,$$

and

$$\frac{L}{R} = 114.0 \text{ for } C = 2.$$

These are the approximate points where columns in actual test cease to follow Euler's law.

Figure 12 is the column strength curves for a proposed standard list of duralumin round tubes (See Table C). Figure 12 was used in plotting these curves. They are plotted only for $C = 1$ because of the difficulty in obtaining any degree of fixity with duralumin tubes in practice.

Figure 13 is the column strength curves for chrome-molybdenum steel streamlined tubes. These tubes are all on the proposed standard list as shown in Table D. Most of these tubes are available from an American steel tube manufacturing concern. The curves are plotted for a fixity coefficient of 1 because

most struts that are in the air stream and are necessarily streamlined are pin-ended.

All these column strength curves are used in the same manner and need very little explanation, but let us take an example. Suppose we have a member in a fuselage structure. The fuselage is entirely welded so that we can safely figure on a fixity of 2 (C = 2). The member is 60 inches long and has a compression load of 18,500 pounds for one condition of design, and a tension load of 26,000 pounds in another condition of design. If we are using chrome-molybdenum steel with a yield point of 60,000 pounds per square inch, we must refer to figure 8. Here we follow up the 60-inch ordinate until we get to the abscissa of 18,500 pounds. We see that for the compression load we have an option of three tubes: 2" × .065", 1-3/4" × .083", or 1-1/2" × .120". The first is the lightest tube, but it will not take the tension load; the second is good for 26,200 pounds tension and will probably be the best to use. However, if for some reason we wanted to conserve space by using a small diameter tube, we could use the 1-1/2" × .120" and have plenty of strength in both tension and compression.

Take another example: Suppose we have a chrome-molybdenum tube welded in at both ends and we want to know its strength in compression. The tube is 1-7/8" o.d. × .058" wall × 55" long. Referring to Figure 8, we see that the tube is good for 16,000 pounds compression. If we take the yield point as the tensile

strength, the tube will be good for 17,150 pounds tension. Now, if we have a good fitting on the end which we think can develop the full strength of the tube, or, say, at least 90,000 pounds per square inch, we can multiply 17,150 by 1.5 or take half of this and add it to it, giving us a tensile strength of 17,150 pounds plus 8,575 pounds equals 25,725 pounds.

Figures 14 and 15 are useful weight curves for round tubing and need no explanation. Note that the weight of chrome-molybdenum streamline tubes can be determined from Figure 14 since the streamline tube has the same cross-sectional area as its basic round tube.

TABLE A

Standard Seamless Round Steel Tubes

for Airplane Structures

Outside diam. Inches		Mild carbon steel Steel #1020 or #1025					Alloy steel (Chrome-molybdenum, 2330 nickel)									
	t	.035	.049	.058	.065	.083	.035	.049	.058	.065	.083	.095	.120	5/32	3/16	1/4
	BWG	20	18	17	16	14	20	18	17	16	14	13	11			
3/16		X														
1/4		X														
5/16		X														
3/8		X														
1/2		X														
5/8		X		X												
3/4		X		X			X									
7/8		X		X			X									
1		X	X	X	X		X	X	X							
1-1/8		X	X	X			X	X	X							
1-1/4		X	X	X	X		X	X	X							
1-3/8			X	X				X	X							
1-1/2			X	X	X			X	X		X	X	X			
1-5/8			X	X				X	X							
1-3/4				X	X			X	X		X	X	X			
1-7/8				X	X				X	X						
2					X	X			X	X	X	X				
2-1/4					X	X				X	X	X	X			
2-1/2						X					X	X				
2-3/4											X	X	X			
3												X	X			
3-1/4													X	X	X	X
3-3/4															X	X

TABLE B

Sample Computations for Column Curves

1-5/8" x .049" #18 R = .557 C = 1 A = .243				1-5/8" x .058" #17 R = .558 C = 1 A = .286				1-3/4" x .049" #18 R = .600 C = 1 A = .263			
L	L/R	P/A	P	L	L/R	P/A	P	L	L/R	P/A	P
0	-	60000	14600	0	-	60000	17150	0	-	60000	15800
10	17.95	58700	14300	10	18.1	58700	16800	10	16.6	58800	15450
20	35.8	56000	13600	20	36.2	55800	15950	20	33.3	56500	14850
30	53.7	51000	12400	30	54.1	50800	14500	30	50.0	52200	13720
40	71.7	43700	10600	40	72.3	43300	12400	40	66.6	45700	12020
50	89.6	34400	8350	50	90.4	33800	9380	50	83.4	37700	9910
60	107.6	24500	5950	60	108.4	24100	6900	60	100.0	28300	7440
70	125.5	18000	4370	70	126.5	17700	5060	70	116.8	20800	5470
80	143.5	13800	3360	80	144.6	13600	3890	80	133.3	16000	4210
90	161.4	10900	2650	90	162.6	10700	3090	90	150.0	12500	3290
100	179.5	8800	2140	100	181.0	8700	2490	100	166.6	10100	2660
110	197.0	7400	1800	110	198.5	7300	2090	110	183.4	8500	2240
120	-	-	-	120	-	-	-	120	200.0	7200	1890

These sample computations are for round chrome-molybdenum tubes with a yield point of 60,000 lb./sq.in.

L = length of strut

R = radius of gyration

P = axial load in pounds

A = area in square inches.

TABLE C

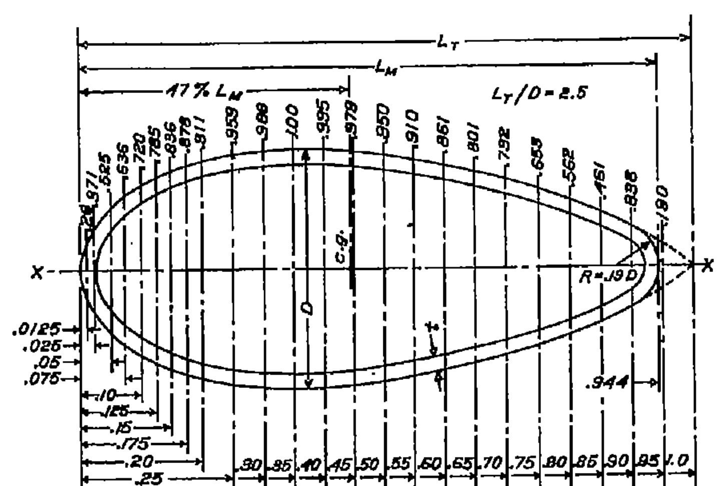
Proposed Standard List of Duralumin Round Tubes

Outside diameter Inches	Wall thickness - Inches and B.W.G.						
	.035 #20	.049 #18	.058 #17	.065 #16	.083 #14	.095 #13	.120 #11
1/2	X						
5/8	X		X				
3/4	X		X				
7/8	X		X				
1	X	X	X	X			
1-1/8	X	X	X				
1-1/4	X	X	X	X			
1-3/8		X	X				
1-1/2		X	X	X	X		
1-5/8		X	X				
1-3/4			X	X	X	X	
1-7/8			X	X			
2			X	X	X	X	
2-1/4					X	X	X
2-1/2				X	X	X	X
2-3/4					X	X	X
3						X	X

TABLE D

Properties of Seamless Streamline Tubes

Basic round tube			Drawn streamline tube				Basic round tube			Drawn streamline tube			
O.D.	Wall thickness	Area	Major axis (L_M)	Minor axis (D)	I_{xx}	R_{xx}	O.D.	Wall thickness	Area	Major axis (L_M)	Minor axis (D)	I_{xx}	R_{xx}
1-1/8	0.035	0.1199	1.517	0.643	0.00608	0.2251	1-7/8	0.058	0.3311	2.528	1.071	0.04664	0.3754
1-1/4	0.035	0.1336	1.685	0.714	0.00841	0.2509		0.065	0.3696			0.05168	0.3739
	0.049	0.1849			0.01139	0.2482	2	0.058	0.3539	2.697	1.143	0.05693	0.4011
1-3/8	0.035	0.1473	1.855	0.786	0.01128	0.2767		0.065	0.3951			0.06313	0.3997
	0.049	0.2041			0.01532	0.2740	2-1/4	0.058	0.3994	3.035	1.286	0.08187	0.4527
1-1/2	0.049	0.2234	2.023	0.857	0.02007	0.2997		0.065	0.4462			0.09088	0.4513
	0.058	0.2628			0.02333	0.2979	2-1/2	0.065	0.4972	3.372	1.429	0.12576	0.5029
1-5/8	0.049	0.2426	2.192	0.929	0.02571	0.3255		0.083	0.6302			0.15710	0.4993
	0.058	0.2855			0.02993	0.3238	2-3/4	0.065	0.5483	3.708	1.571	0.16859	0.5545
1-3/4	0.049	0.2618	2.360	1.000	0.03232	0.3514		0.083	0.6954			0.21104	0.5509
	0.058	0.3083			0.03767	0.3496	3	0.065	0.5993	4.045	1.714	0.22018	0.6061
	0.065	0.3441			0.04170	0.3481		0.083	0.7606			0.27611	0.6025
1-7/8	0.049	0.2811	2.528	1.071	0.04073	0.3807							



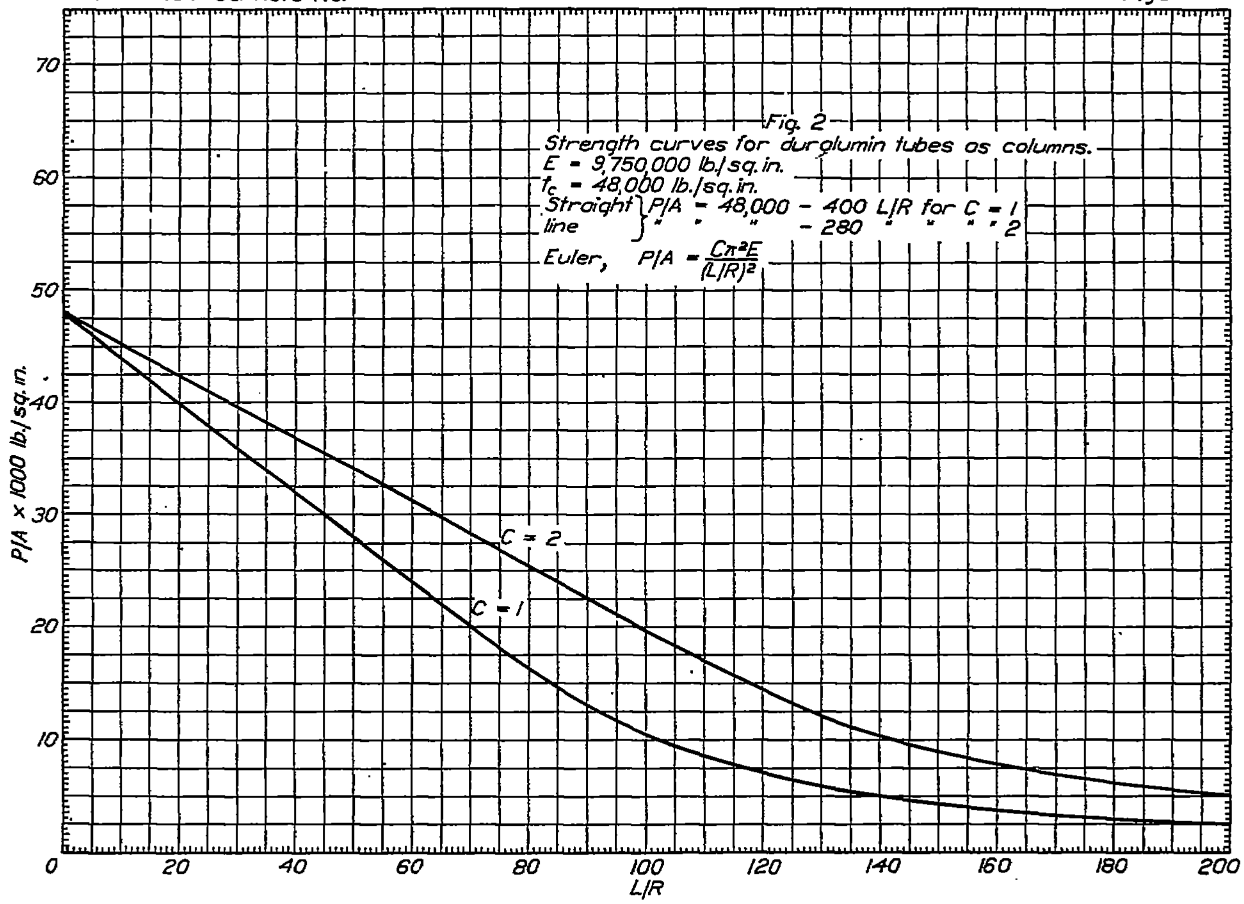


Fig. 2
Strength curves for duralumin tubes as columns.
E = 9,750,000 lb./sq.in.
fc = 48,000 lb./sq.in.
Straight line } P/A = 48,000 - 400 L/R for C = 1
" " " - 280 " " " 2
Euler, P/A = Cπ²E/(L/R)²
C = 2
C = 1
P/A × 1000 lb./sq.in.
L/R
0
10
20
30
40
50
60
70
80
100
120
140
160
180
200

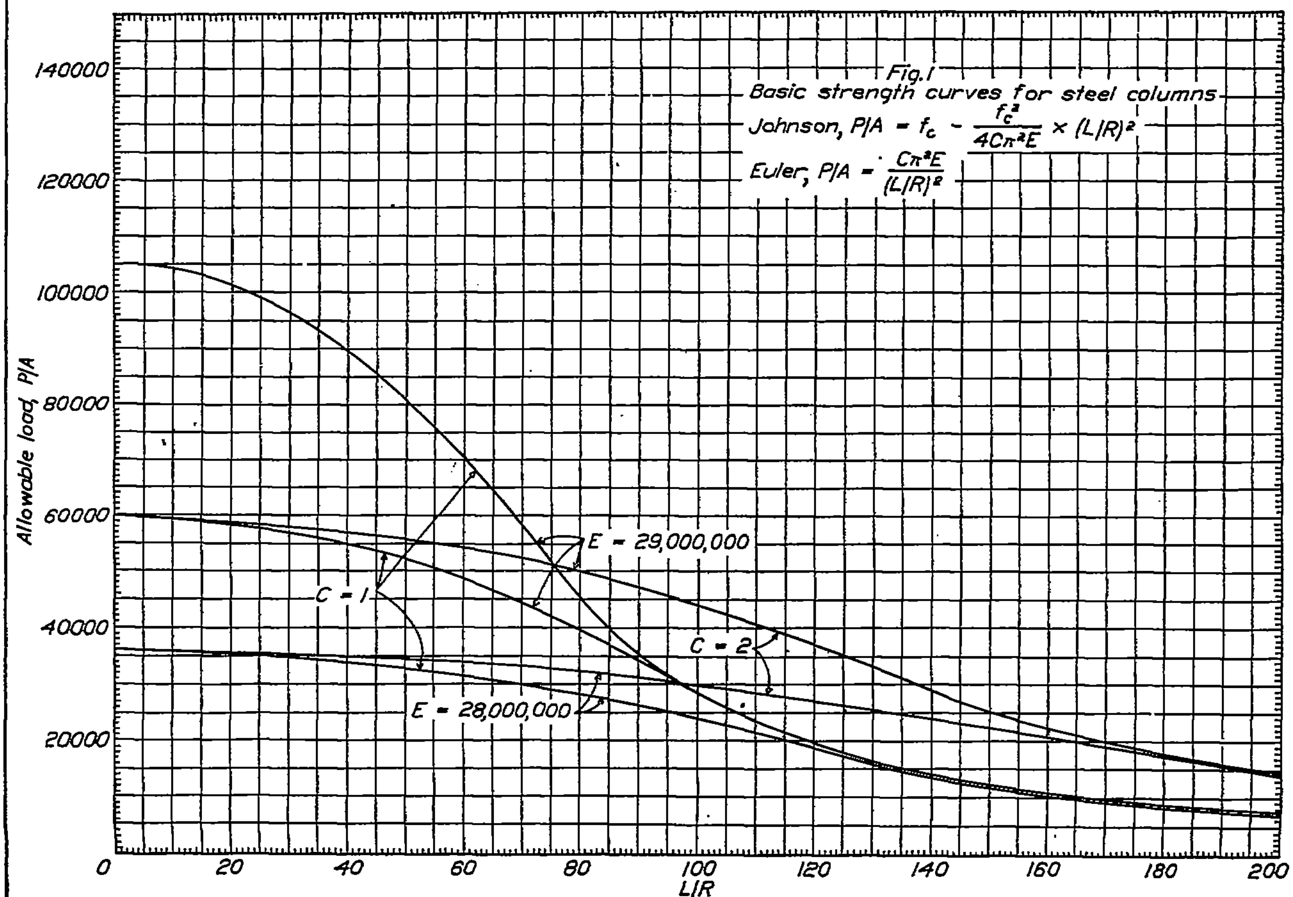


Fig. 1
Basic strength curves for steel columns
Johnson, P/A = fc - fc²/4Cπ²E × (L/R)²
Euler, P/A = Cπ²E/(L/R)²
E = 29,000,000
E = 28,000,000
C = 1
C = 2
Allowable load, P/A
L/R
20000
40000
60000
80000
100000
120000
140000
0
20
40
60
80
100
120
140
160
180
200

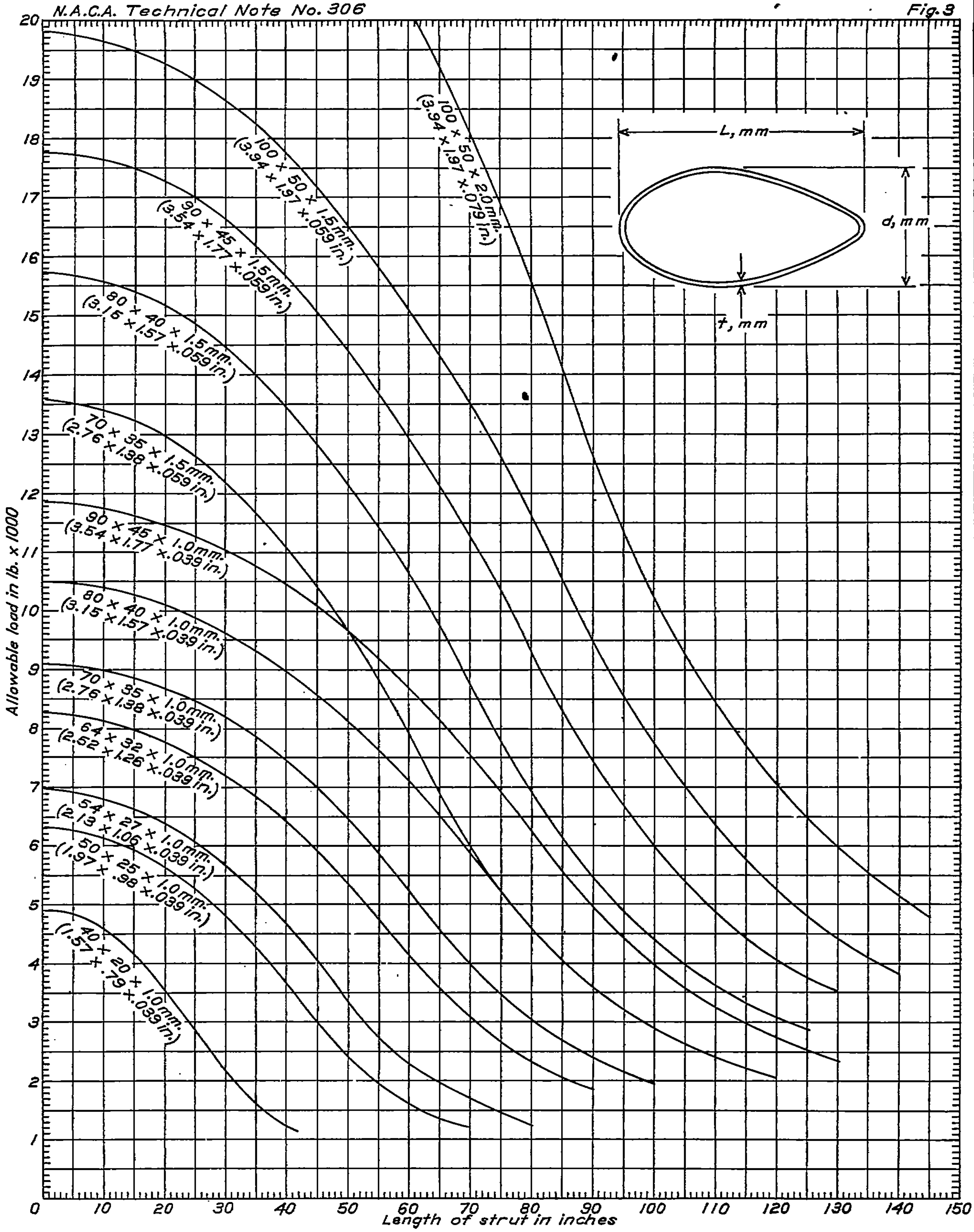


Fig. 3 Strength curves for carbon steel streamline tubes as columns.
Y.P. in compression = 36,000 lb./sq.in.
Fixity coef. = C = 1 E = 28,000,000 lb./sq.in.

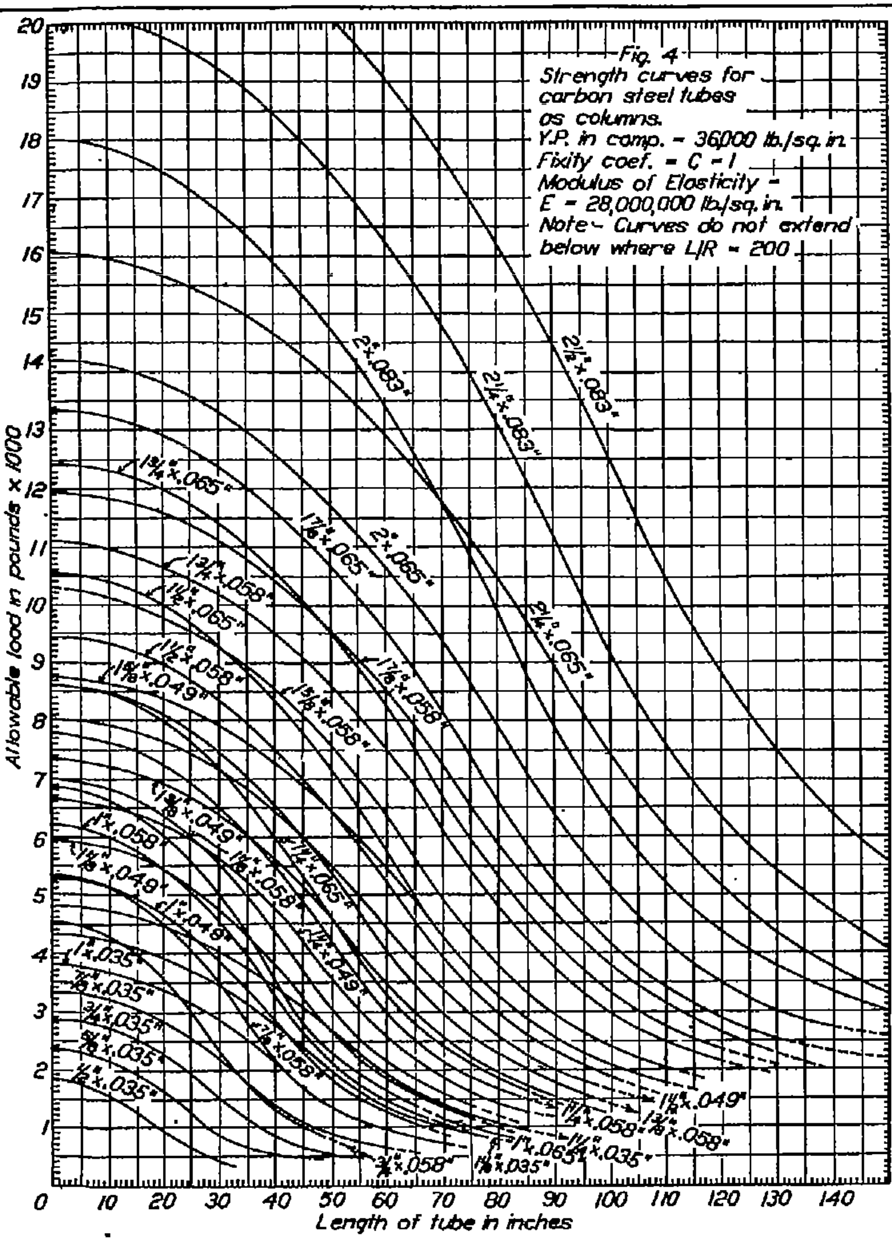


Fig. 4
Strength curves for carbon steel tubes as columns.
Y.P. in comp. = 36,000 lb./sq.in.
Fixity coef. = C = 1
Modulus of Elasticity = E = 28,000,000 lb./sq.in.
Note - Curves do not extend below where L/R = 200

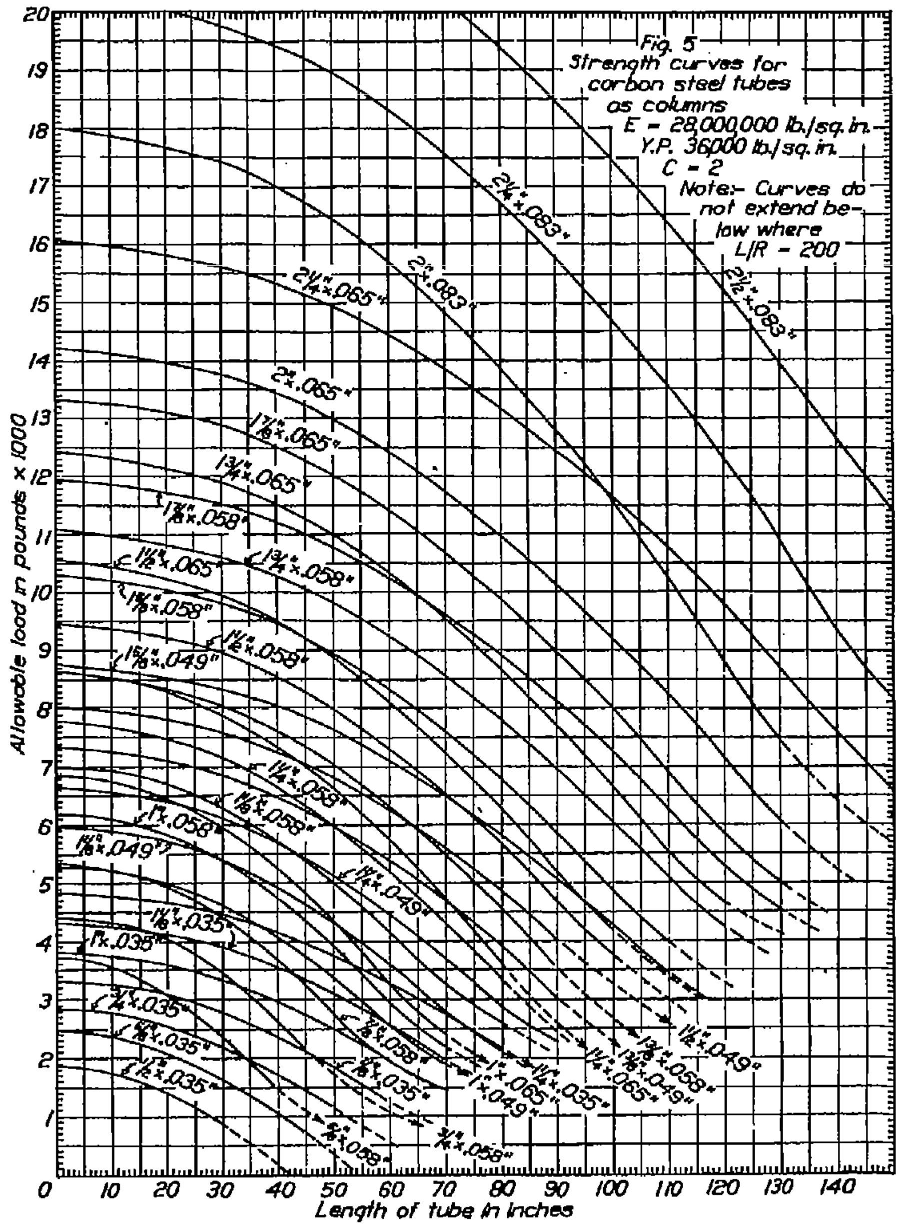
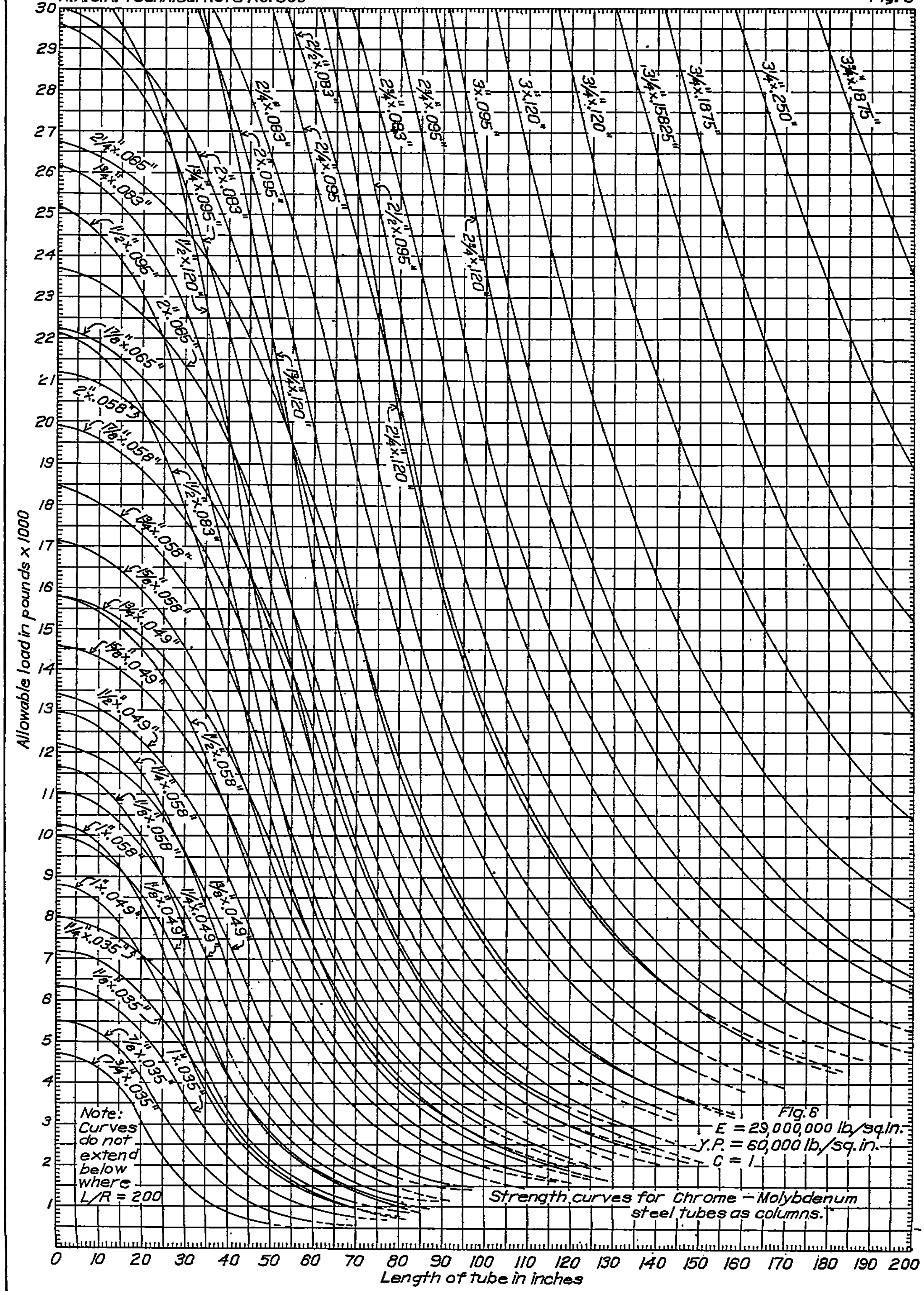


Fig. 5
Strength curves for carbon steel tubes as columns
E = 28,000,000 lb./sq.in.
Y.P. 36,000 lb./sq.in.
C = 2
Note:- Curves do not extend below where L/R = 200



Allowable load in pounds x 1000
Length of tube in inches
Note: Curves do not extend below where L/R = 200
Fig. 6
E = 29,000,000 lb/sq.in.
Y.P. = 60,000 lb/sq.in.
C = 1
Strength curves for Chrome - Molybdenum steel tubes as columns.

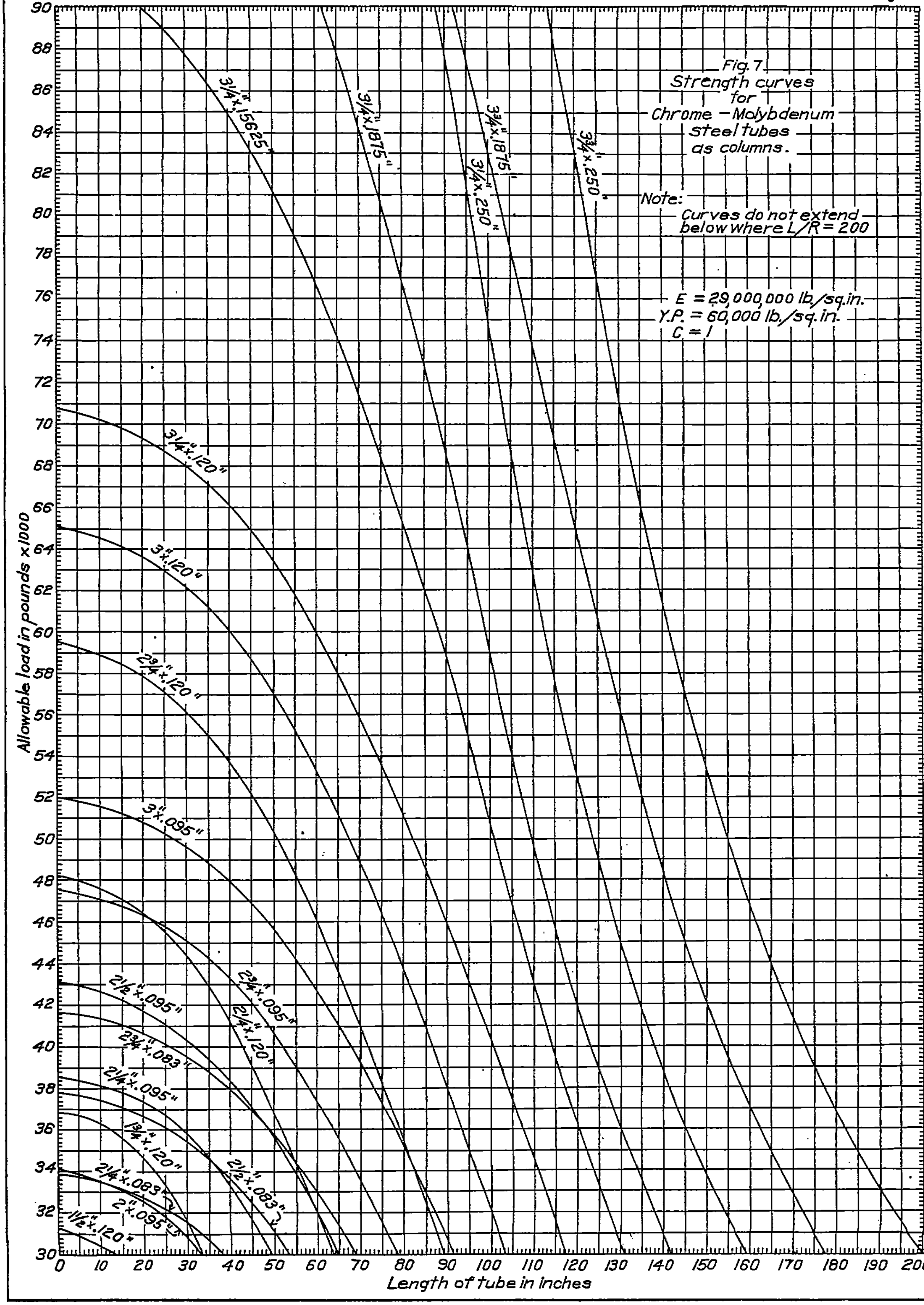


Fig. 7
Strength curves for Chrome - Molybdenum steel tubes as columns.
Note: Curves do not extend below where L/R = 200
E = 29,000,000 lb./sq.in.
Y.P. = 60,000 lb./sq.in.
C = 1
3 1/4" x .15625"
3 1/4" x .1875"
3 3/4" x .1875"
3 1/4" x .250"
3 3/4" x .250"
3 1/4" x .120"
3" x .120"
2 3/4" x .120"
3" x .095"
2 3/4" x .095"
2 1/4" x .120"
2 1/2" x .095"
2 3/4" x .083"
2 1/4" x .095"
1 3/4" x .120"
2 1/2" x .083"
2 1/4" x .083"
2" x .095"
1 1/2" x .120"
Allowable load in pounds x 1000
Length of tube in inches

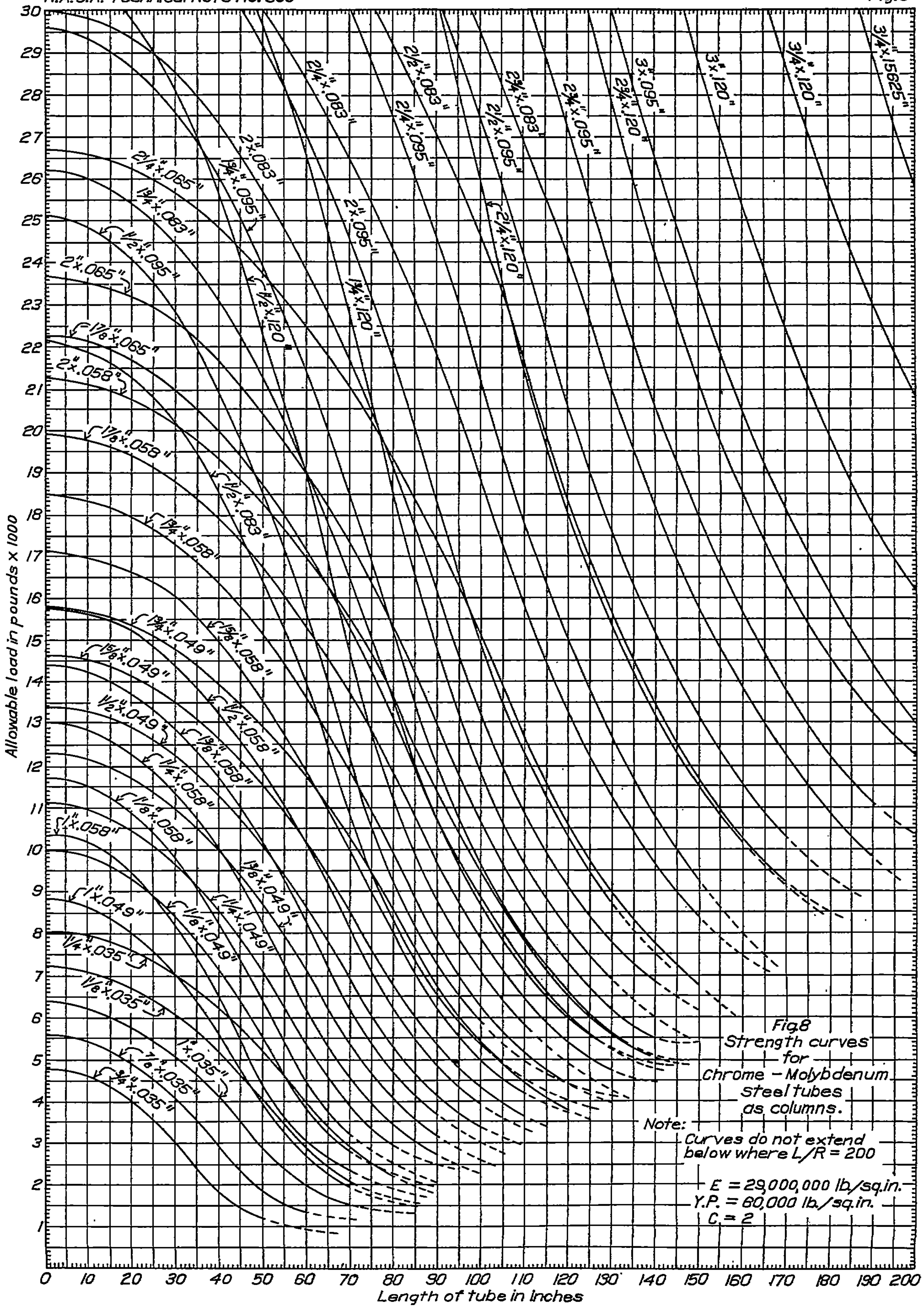


Fig. 8 Strength curves for Chrome - Molybdenum steel tubes as columns.

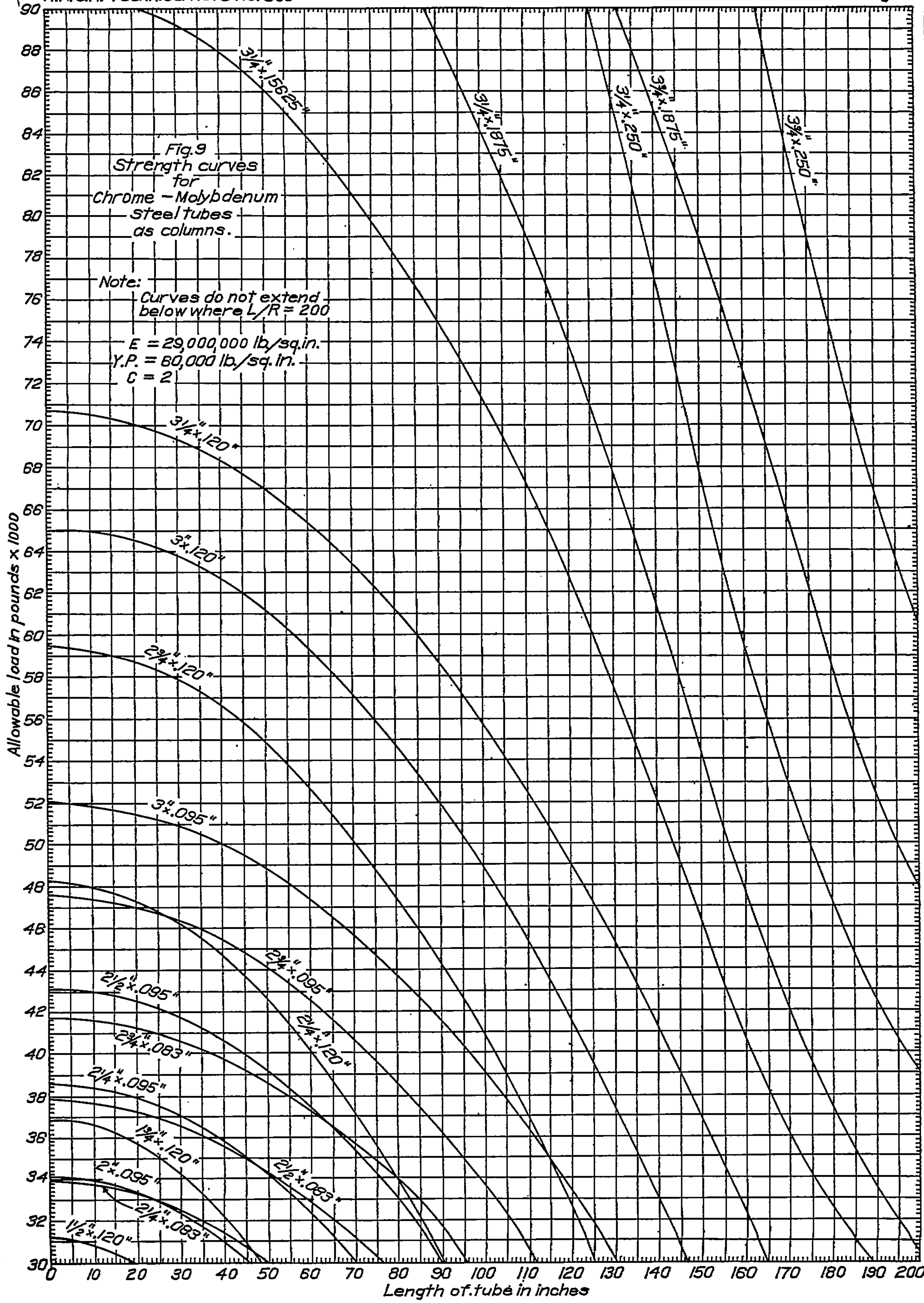


Fig. 9
Strength curves
for
Chrome - Molybdenum
steel tubes
as columns.
Note:
Curves do not extend
below where L/R = 200
E = 29,000,000 lb./sq.in.
Y.P. = 60,000 lb./sq.in.
C = 2
3¼" × .15625"
3¼" × .1875"
3¼" × .250"
3¾" × .1875"
3¾" × .250"
3¼" × .120"
3" × .120"
2¾" × .120"
3" × .095"
2¾" × .095"
2¼" × .120"
2½" × .095"
2¾" × .083"
2¼" × .095"
1¾" × .120"
2½" × .083"
2" × .095"
2¼" × .083"
1½" × .120"
Allowable load in pounds × 1000
Length of tube in inches
90
88
86
84
82
80
78
76
74
72
70
68
66
64
62
60
58
56
54
52
50
48
46
44
42
40
38
36
34
32
30
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200

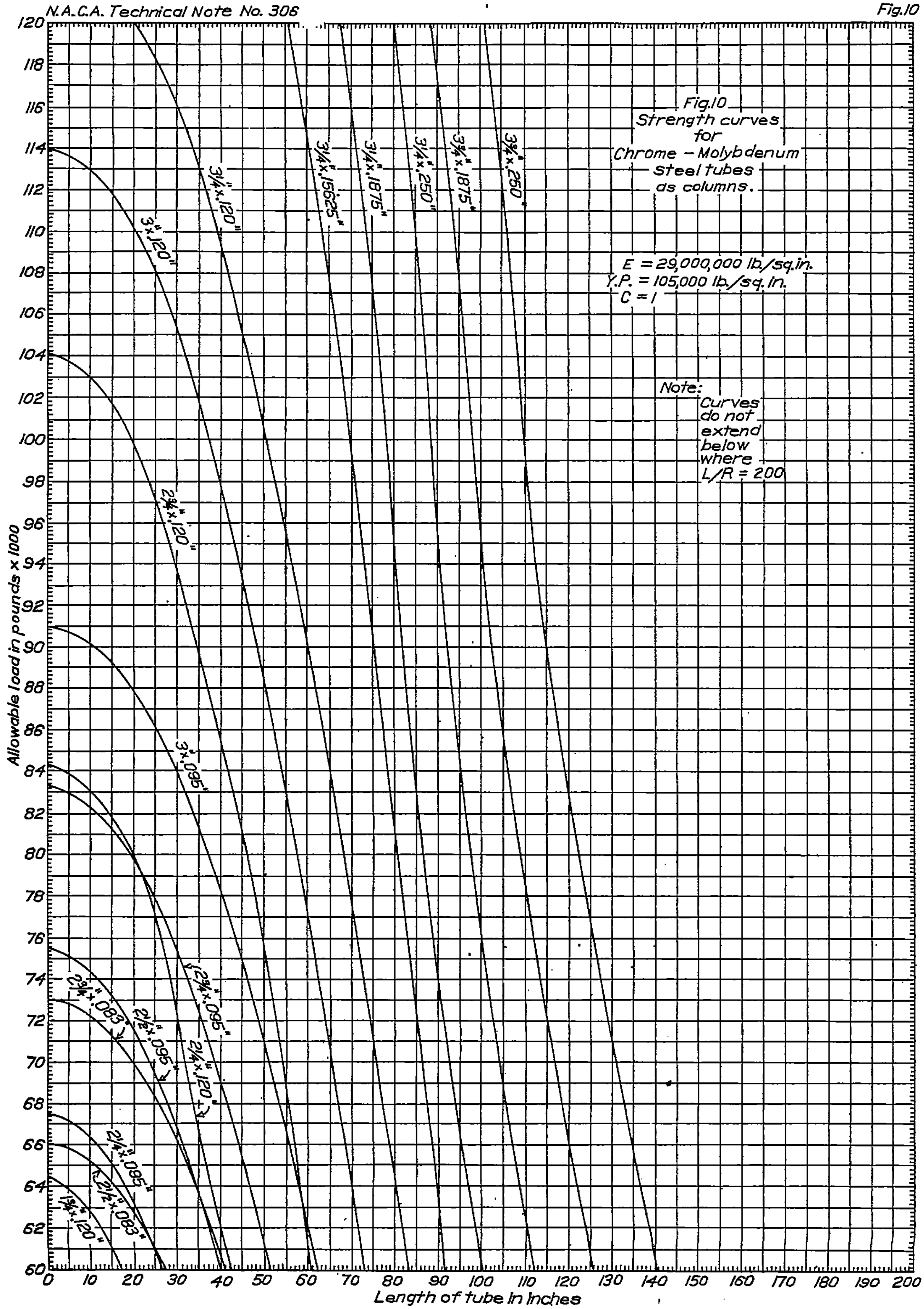


Fig.10
Strength curves
for
Chrome - Molybdenum
steel tubes
as columns.
E = 29,000,000 lb/sq.in.
Y.P. = 105,000 lb/sq.in.
C = 1
Note:
Curves do not extend below where L/R = 200
3¼"x.120"
3"x.120"
3¼"x.15625"
3¼"x.1875"
3¼"x.250"
3¾"x.1875"
3¾"x.250"
2¾"x.120"
3"x.095"
2¾"x.095"
2¼"x.083"
2½"x.095"
2¼"x.120"
2¼"x.095"
2½"x.083"
1¾"x.120"
Allowable load in pounds x 1000
Length of tube in inches

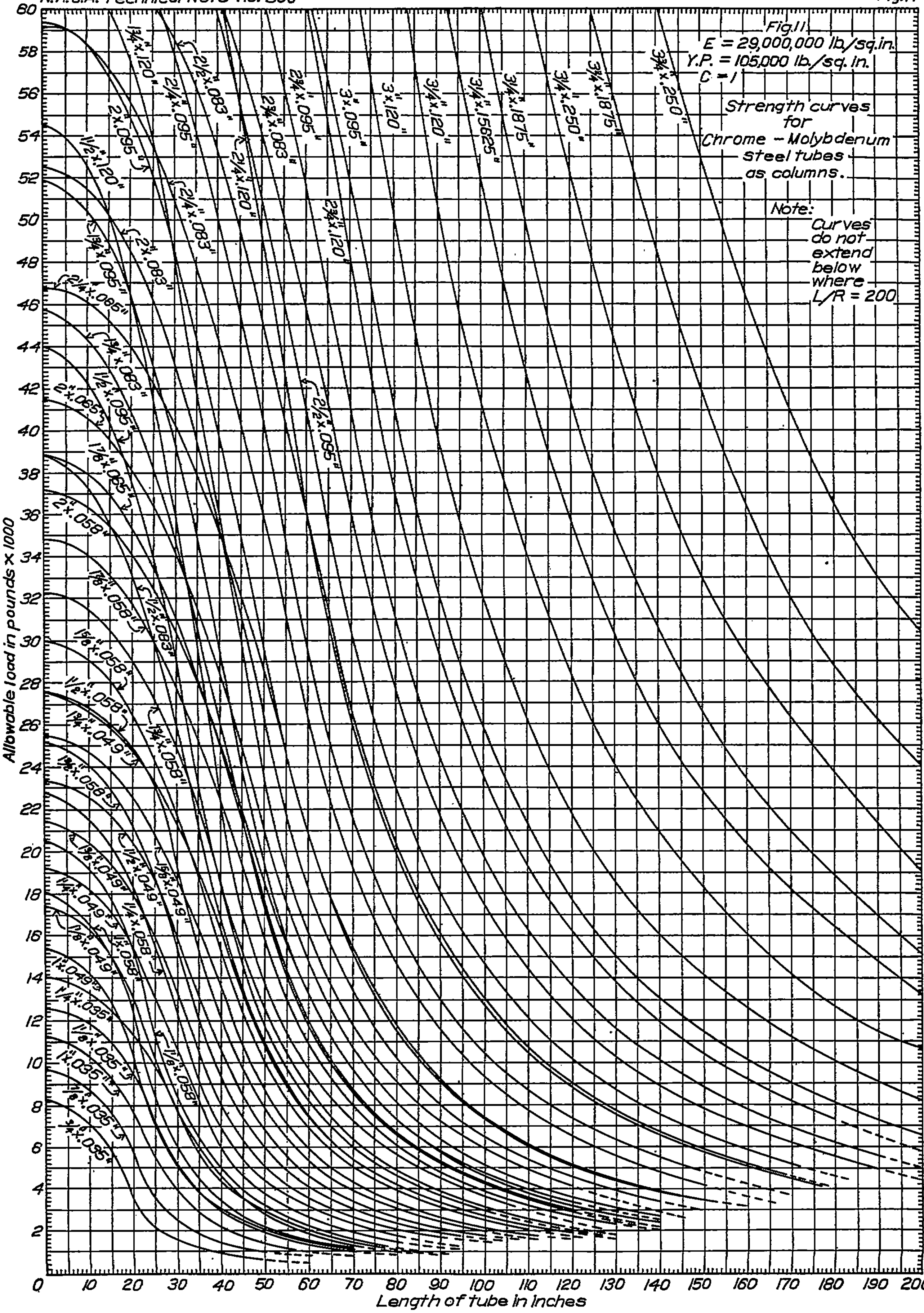


Fig.11
E = 29,000,000 lb/sq.in.
Y.P. = 105,000 lb./sq.in.
C = 1
Strength curves for Chrome - Molybdenum steel tubes as columns.
Note: Curves do not extend below where L/R = 200
Allowable load in pounds × 1000
Length of tube in inches

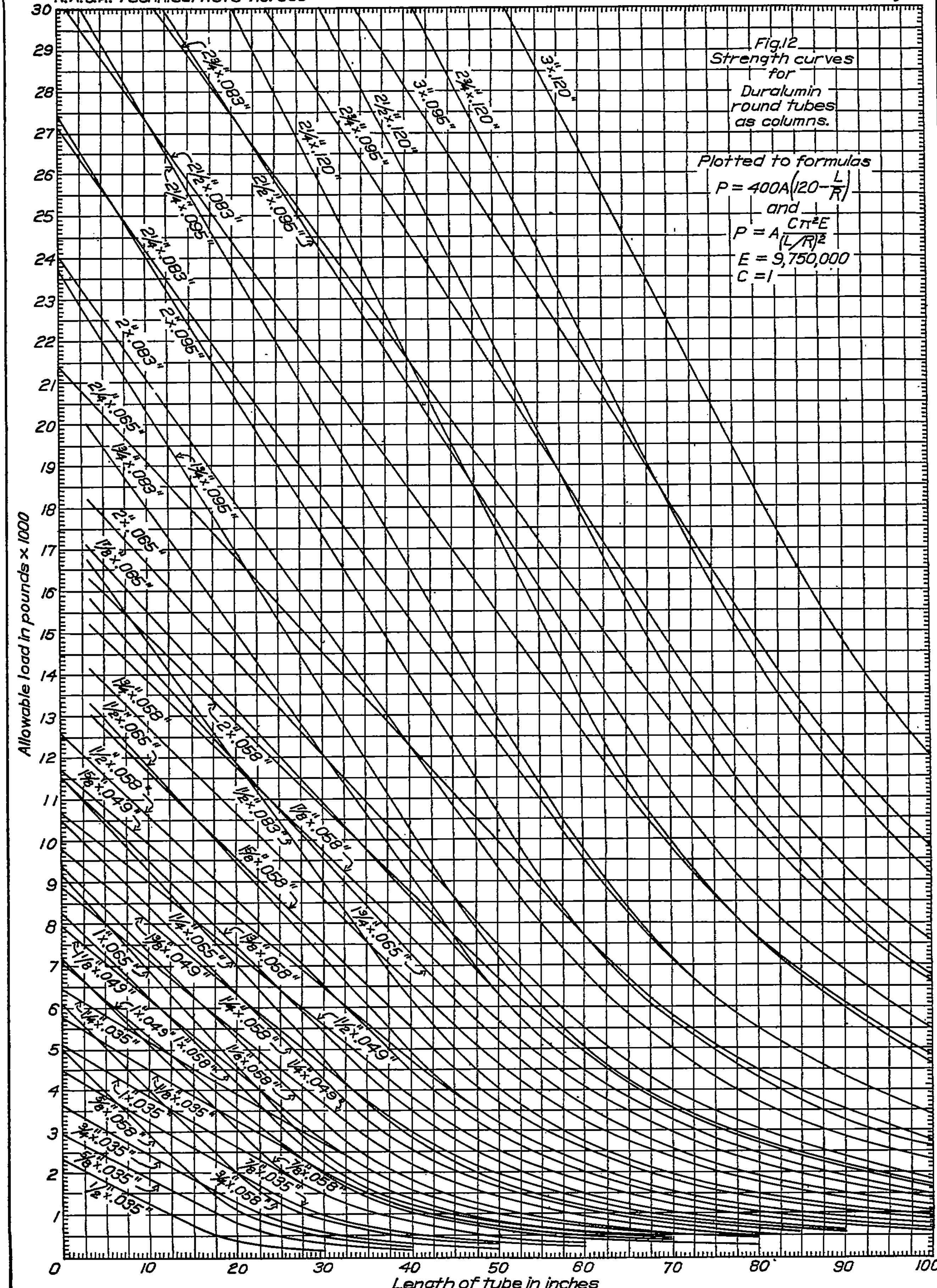


Fig.12
Strength curves for Duralumin round tubes as columns.
Plotted to formulas
$P = 400A\left(120 - \frac{L}{R}\right)$
and
$P = A\frac{C\pi^2 E}{(L/R)^2}$
$E = 9,750,000$
$C = 1$
3"x.120"
2¾"x.120"
3"x.095"
2½"x.120"
2¾"x.095"
2¼"x.120"
2¾"x.083"
2½"x.095"
2½"x.083"
2¼"x.095"
2¼"x.083"
2"x.095"
2"x.083"
2¼"x.065"
1¾"x.083"
1¾"x.095"
2"x.065"
1⅞"x.065"
1¾"x.058"
1½"x.065"
2"x.058"
1½"x.058"
1⅝"x.049"
1½"x.083"
1⅞"x.058"
1⅝"x.058"
1¾"x.065"
1¼"x.065"
1⅜"x.049"
1⅜"x.058"
1"x.065"
1⅛"x.049"
1"x.049"
1¼"x.058"
1½"x.049"
1¼"x.035"
1"x.058"
1⅛"x.058"
1¼"x.049"
1"x.035"
1⅛"x.035"
⅞"x.058"
¾"x.035"
⅞"x.035"
⅞"x.035"
⅞"x.058"
¾"x.058"
½"x.035"
Allowable load in pounds x 1000
Length of tube in inches
0 10 20 30 40 50 60 70 80 90 100
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

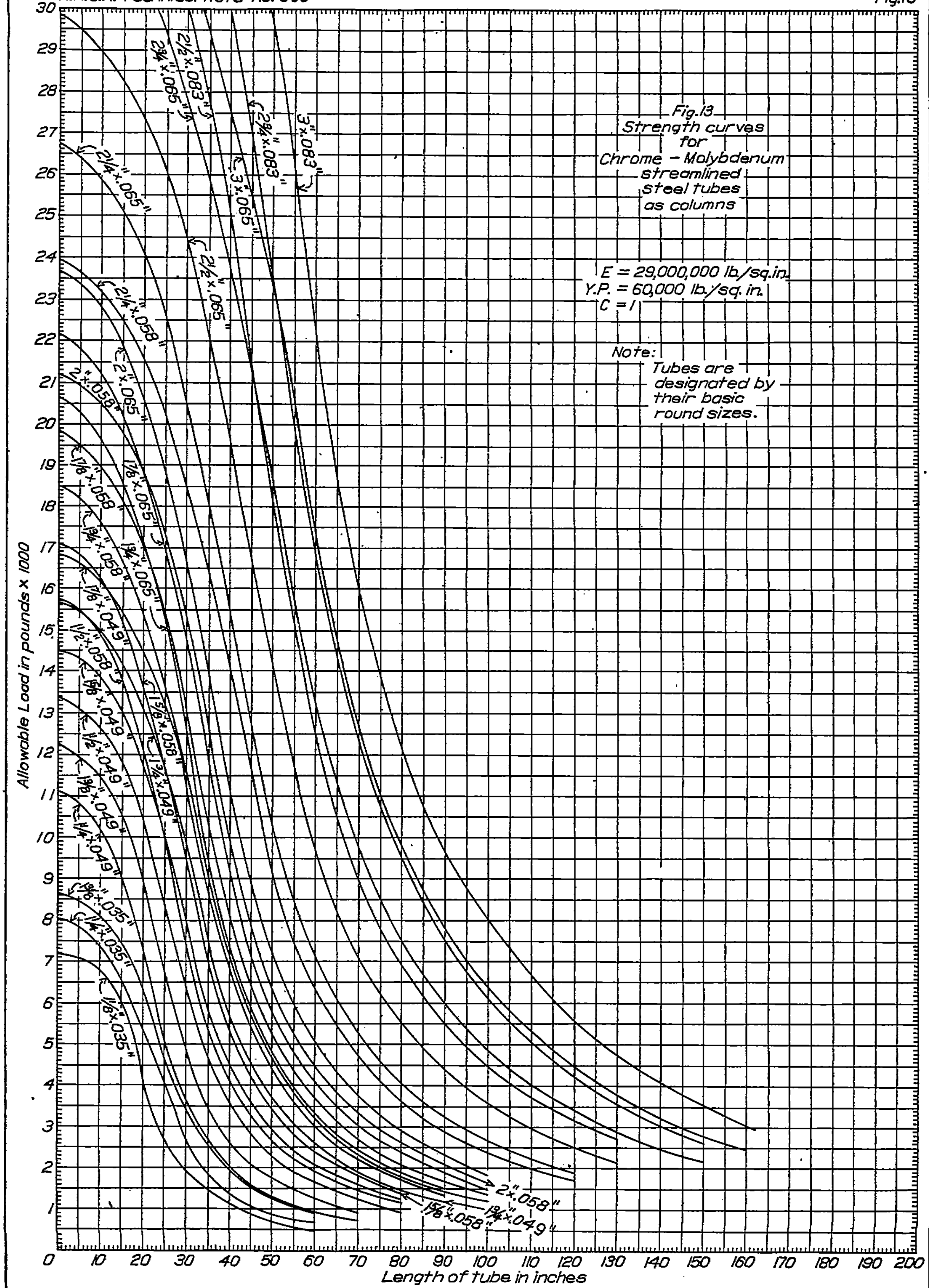


Fig.13
Strength curves for Chrome - Molybdenum streamlined steel tubes as columns
E = 29,000,000 lb/sq.in.
Y.P. = 60,000 lb/sq.in.
C = 1
Note: Tubes are designated by their basic round sizes.
Allowable Load in pounds x 1000
Length of tube in inches
3" x .083"
2 3/4" x .083"
2 1/2" x .083"
2 3/4" x .065"
3" x .065"
2 1/2" x .065"
2 1/4" x .065"
2 1/4" x .058"
2" x .065"
2" x .058"
1 7/8" x .065"
1 7/8" x .058"
1 3/4" x .065"
1 3/4" x .058"
1 5/8" x .058"
1 5/8" x .049"
1 1/2" x .058"
1 3/4" x .049"
1 1/2" x .049"
1 3/8" x .049"
1 1/4" x .049"
1 3/8" x .035"
1 1/4" x .035"
1 1/8" x .035"
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

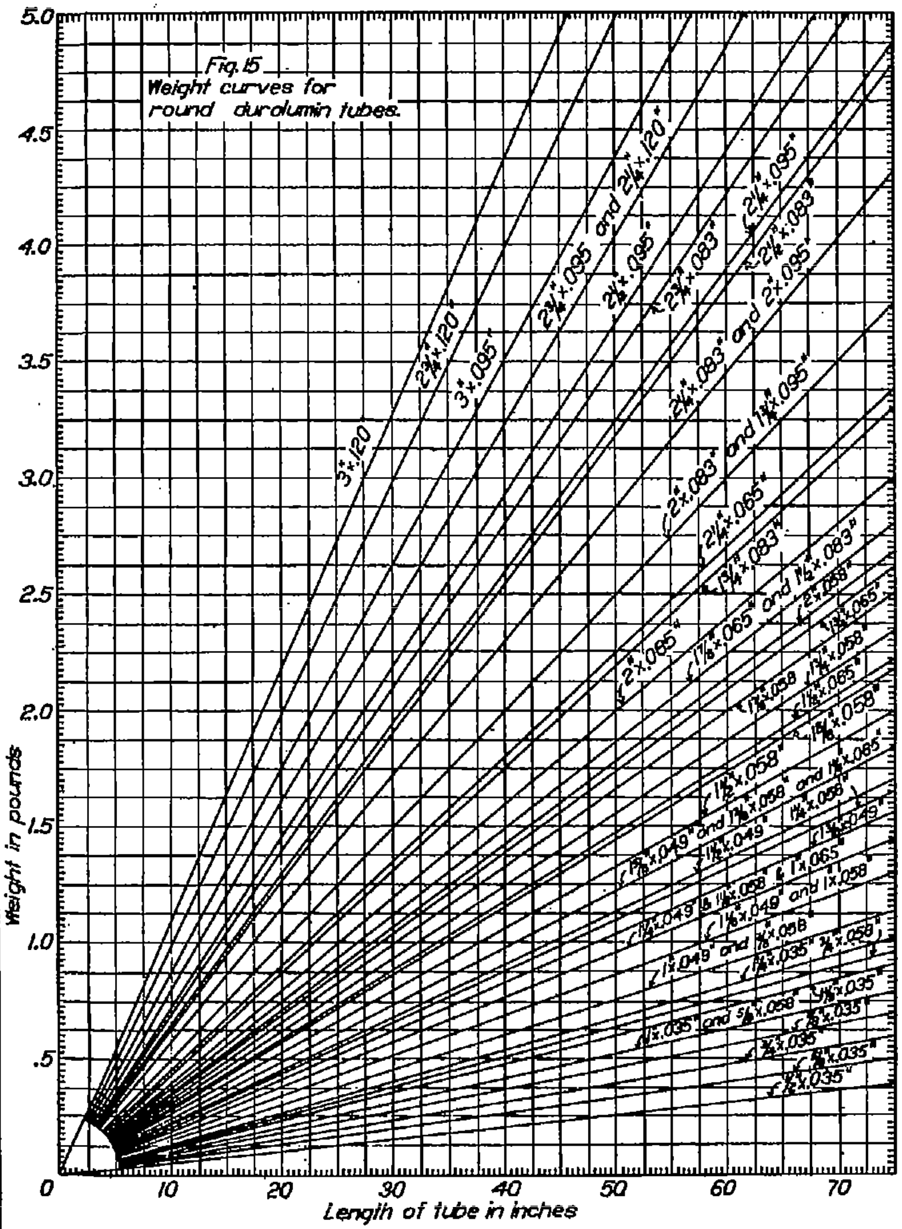


Fig. 15
Weight curves for round duralumin tubes.

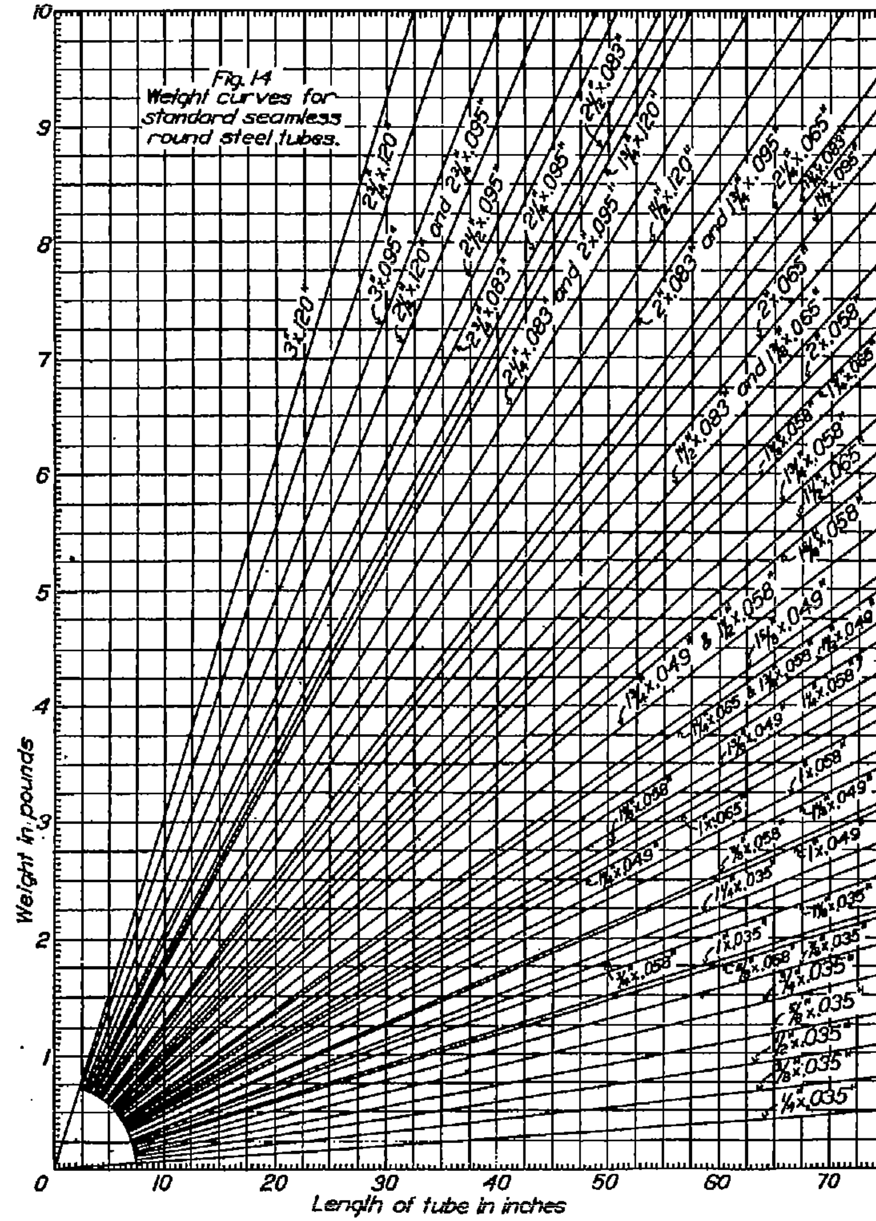


Fig. 14
Weight curves for standard seamless round steel tubes.